

# A Comment on the Radiative Decays of the $\Lambda(1405)$

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## Abstract

We examine the radiative decays of the  $\Lambda(1405)$  to octet baryons,  $\Lambda(1405) \rightarrow \Lambda\gamma$  and  $\Lambda(1405) \rightarrow \Sigma^0\gamma$ . In the limit of SU(3) symmetry the decay rates for these two modes are related, and we compute the leading correction to the relation in chiral perturbation theory. Such measurements will allow the sign of the strong coupling  $g_{\Lambda^*}$  to be determined. The SU(3) violating radiative decay to the baryon decuplet  $\Lambda(1405) \rightarrow \Sigma^{*0}\gamma$  dominated by one-loop graphs is also discussed. Experiments planned for the Jefferson Laboratory should be able to measure some, if not all, of the processes considered.

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The  $\Lambda(1405)$  is an interesting object from the standpoint of hadronic structure. Lying 30 MeV below the  $NK$  threshold, it has long been thought to be a “bound state” of a nucleon and a kaon. In contrast, the most naive constituent quark model describes the  $\Lambda(1405)$  as mainly a  $SU(3)$  singlet with small admixtures of octet. In reality, the actual “structure” of this hadron will lie somewhere between these two extreme pictures. Its intrinsic hybrid nature poses problems for the standard hadronic tools and models with the small energy scale of 30 MeV giving rise to severe convergence problems. In order to have any confidence in our understanding of this baryon it is necessary to make comparisons between experimental measurements and theoretical predictions for observables that probe different aspects of its structure. The strong interactions of this object have been considered extensively in the past. In chiral perturbation theory where the  $\Lambda(1405)$  is treated as an  $SU(3)$  singlet the close proximity of the  $KN$  threshold gives rise to large uncertainties in the loop-level amplitudes [1,2]. Such problems are sometimes not observable in model computations as specific assumptions are usually made about the dynamics.

Matrix elements of the electromagnetic current provide a direct measure of the charge and current distributions of a hadron. One suspects that the computation of the radiative decays of the  $\Lambda(1405)$  will encounter the same technical difficulties that plague the computation of its strong interactions. The potential problem can be clearly seen in the picture where the  $\Lambda(1405)$  is treated as a  $NK$  bound state. The characteristic length scale for observables will generally be set by the radius of the bound state, determined by the binding energy, as opposed to the chiral symmetry breaking scale. Such a configuration is expected to give rise to large  $E1$  matrix elements for radiative transitions to the octet baryons. Consequently, how such an extended configuration is “handled” in a given computational scheme may strongly influence the predictions for the radiative widths. Such model dependent predictions have been made for the widths of the radiative decays  $\Lambda(1405) \rightarrow \Lambda\gamma$  and  $\Lambda(1405) \rightarrow \Sigma^0\gamma$  with different models disagreeing significantly with each other [3–9] .

In the limit of exact  $SU(3)$  flavor symmetry and assuming the  $\Lambda(1405)$  to be an  $SU(3)$  singlet, there is a relation between the radiative decay widths. While we cannot compute the individual decay rates directly from QCD we can compute the leading corrections to the flavor symmetry relation using chiral perturbation theory. Previously, an analysis of kaon photo-production and radiative capture has been used to extract a measure of  $SU(3)$  breaking in these decays [10] , however, the results appear to be model dependent. It is with an eye to the future of high precision measurements of the radiative branching fractions of the  $\Lambda(1405)$  that we re-examine these decays. The CEBAF experiment E89-024 [11] will make a precise determination of the branching fraction for  $\Lambda(1405) \rightarrow \Lambda\gamma$ , detecting  $\sim 10^3$  events, but a precise measurement of the  $\Lambda(1405) \rightarrow \Sigma^0\gamma$  branching fraction will be required to make use of the work in this paper.

The  $\Lambda(1405)$  will be treated as an  $SU(3)$  singlet object with spin and parity  $J^\pi = \frac{1}{2}^-$ . The lagrange density describing its interactions, along with those of the lowest lying baryon octet and the pseudo-Goldstone bosons is given at leading order in the chiral expansion by [1,2,12]

$$\begin{aligned} \mathcal{L} = & Tr [\bar{B} i v \cdot DB] + \bar{\Lambda}^* i v \cdot \partial \Lambda^* - \Delta_\Lambda^* \bar{\Lambda}^* \Lambda^* \\ & + 2D Tr [\bar{B} S^\mu \{A_\mu, B\}] + 2F Tr [\bar{B} S^\mu [A_\mu, B]] + g_\Lambda^* (\bar{\Lambda}^* Tr [v \cdot AB] + h.c.) \quad , \quad (1) \end{aligned}$$

where  $A_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi)$  is the axial meson field with  $\xi = \exp(iM/f)$  and  $f =$

132 MeV is the meson decay constant. The chiral covariant derivative is  $D_\mu B = \partial_\mu B + [V_\mu, B]$  with  $V_\mu = \frac{1}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi)$  and the baryon octet field is given by

$$B = \begin{pmatrix} \Lambda/\sqrt{6} + \Sigma^0/\sqrt{2} & \Sigma^+ & p \\ \Sigma^- & \Lambda/\sqrt{6} - \Sigma^0/\sqrt{2} & n \\ \Xi^- & \Xi^0 & -2\Lambda/\sqrt{6} \end{pmatrix} \quad (2)$$

We have not included the mesonic interactions between decuplet baryons and the  $\Lambda(1405)$  as the meson and the decuplet baryons must be in a relative D-wave. Such interactions can only contribute to the radiative decays at higher order than we are working. The axial coupling constants  $F$  and  $D$  have been determined from semileptonic decays of the octet baryons and we use  $F = 0.4$  and  $D = 0.6$  [13] for our calculations (the difference between the loop-level values and the tree-level values is formally higher order in the chiral expansion). The coupling of the  $\Lambda(1405)$  to the octet baryons,  $g_{\Lambda^*}$  is determined from the width of the  $\Lambda(1405)$  and at loop level it is found to be  $|g_{\Lambda^*}| = 0.40 \pm 0.04$  [2] .

The radiative decay of the  $\Lambda(1405)$  to the lowest lying octet baryons proceeds by E1 radiation. At lowest order in the chiral expansion the matrix element for this decay receives a contribution from the dim-5 operator

$$\begin{aligned} \mathcal{L}_{\text{E1}}^{(0)} &= \sqrt{6} \kappa^{(0)} \frac{e}{M_N} \bar{B}_b^a Q_a^b \sigma^{\mu\nu} \gamma_5 \Lambda^* F_{\mu\nu} \\ &= \kappa^{(0)} \frac{e}{M_N} \left[ \bar{\Lambda} \sigma^{\mu\nu} \gamma_5 \Lambda^* F_{\mu\nu} + \sqrt{3} \bar{\Sigma}^0 \sigma^{\mu\nu} \gamma_5 \Lambda^* F_{\mu\nu} \right] , \end{aligned} \quad (3)$$

where  $Q = \text{diag}(2/3, -1/3, -1/3)$  is the light quark electromagnetic charge matrix. We have removed a factor of  $M_N$ , the nucleon mass, from the definition of  $\kappa$  for convenience only. Writing the full amplitudes for these decays as

$$\begin{aligned} \mathcal{A}^\Lambda &= i \kappa_\Lambda \frac{e}{M_N} \bar{\Lambda} \sigma^{\mu\nu} \gamma_5 \Lambda^* F_{\mu\nu} \\ \mathcal{A}^\Sigma &= i \kappa_\Sigma \frac{e}{M_N} \bar{\Sigma} \sigma^{\mu\nu} \gamma_5 \Lambda^* F_{\mu\nu} , \end{aligned} \quad (4)$$

one sees that at lowest order there is a SU(3) relation between the matrix elements

$$\kappa_\Lambda = \frac{1}{\sqrt{3}} \kappa_\Sigma . \quad (5)$$

The decay rate for the E1 transition in terms of the  $\kappa_B$  is

$$\Gamma(\Lambda(1405) \rightarrow B\gamma) = |\kappa_B|^2 \frac{4e^2}{\pi M_N^2} E_\gamma^3 . \quad (6)$$

Unfortunately, we are not in a position to compute  $\kappa^{(0)}$ , however we can compute the leading contributions to  $\kappa_\Lambda$  and  $\kappa_\Sigma$  arising from the mass difference between the strange and up, down quark masses. Writing

$$\begin{aligned} \kappa_\Lambda &= \kappa_\Lambda^{(0)} + \kappa_\Lambda^{(1)} + \dots \\ \kappa_\Sigma &= \sqrt{3} \kappa_\Lambda^{(0)} + \kappa_\Sigma^{(1)} + \dots , \end{aligned} \quad (7)$$

we compute the leading contributions to  $\kappa_\Lambda^{(1)}$  and  $\kappa_\Sigma^{(1)}$  from one loop graphs involving the octet baryons and the pseudo-Goldstone bosons. These loop contributions formally dominate over the local counterterms involving insertions of the light quark mass matrix in the chiral limit. It is useful to form the ratio

$$\mathcal{R} = \frac{|\kappa_\Sigma|^2 - 3|\kappa_\Lambda|^2}{\sqrt{3}\sqrt{|\kappa_\Sigma|^2 + |\kappa_\Lambda|^2}} \rightarrow \text{Re}(\kappa_\Sigma^{(1)} - \sqrt{3}\kappa_\Lambda^{(1)}) + \mathcal{O}((\kappa^{(1)})^2/\kappa^{(0)}) \quad , \quad (8)$$

for which a deviation from zero is a direct measure of the magnitude of SU(3) breaking. In forming this ratio we have assumed that the usual power counting for chiral perturbation theory holds, i.e. we have assumed that the loop contribution is subleading compared to the contribution from the incalculable dim-5 operator,  $\kappa^{(0)}$ , but large compared to the contribution from dim-6 operators. This definition of  $\mathcal{R}$  is similar to that of [9,10] however, we have divided through by the denominator so that at leading order  $\mathcal{R}$  is insensitive to the lowest order incalculable counterterm.

Explicit computation of the loop graphs shown in Fig. 1, leads to

$$\begin{aligned} \kappa_\Lambda^{(1)} &= \frac{g_{\Lambda^*} M_N}{16\pi^2 f^2} \left[ \sqrt{\frac{3}{2}} F(G(\Xi, K) + G(N, K)) + \frac{1}{\sqrt{6}} D(G(N, K) - G(\Xi, K)) \right] \\ \kappa_\Sigma^{(1)} &= \frac{g_{\Lambda^*} M_N}{16\pi^2 f^2} \left[ \sqrt{\frac{1}{2}} F(G(\Xi, K) + G(N, K) + 4G(\Sigma, \pi)) - \frac{1}{\sqrt{2}} D(G(N, K) - G(\Xi, K)) \right] \quad , \quad (9) \end{aligned}$$

where the function  $G(B, M)$  is given by

$$G(B, M) = \Gamma(\epsilon) \left[ I(-\epsilon; -\Delta_B, M_M^2) - \int_0^1 dx I(-\epsilon; xE_\gamma - \Delta_B, M_M^2) \right] \quad , \quad (10)$$

where  $\Delta_B = M_{\Lambda^*} - M_B$  and we use  $\overline{MS}$  to regulate the divergence. The function  $I(-\epsilon; b, c)$  is [12]

$$I(-\epsilon; b, c) = \frac{1}{1-2\epsilon} \left[ -bc^{-\epsilon} - \epsilon\sqrt{b^2 - c} \log \left( \frac{b - \sqrt{b^2 - c + i\epsilon}}{b + \sqrt{b^2 - c + i\epsilon}} \right) \right] \quad . \quad (11)$$

The loop contributions have  $\Gamma(\epsilon)$  divergences which are removed by a renormalization of the dim-5 and dim-6 operators but in the limit that the octet baryons are degenerate the SU(3) breaking quantity  $\mathcal{R}$  receives only a finite contribution. When the formally higher order octet mass splittings are retained, one finds that terms of the form  $m_s \log m_s$  are present. We renormalize at the chiral symmetry breaking scale  $\Lambda_\chi$  and do not include the formally subleading counterterms of order  $\mathcal{O}(m_s)$ . The explicit expression for  $\mathcal{R}$  is

$$\begin{aligned} \mathcal{R} &= \frac{g_{\Lambda^*}}{16\pi^2 f_K^2} \frac{M_N}{\sqrt{2}} \left( F \left[ G^\Sigma(\Xi, K) + G^\Sigma(N, K) + 4G^\Sigma(\Sigma, \pi) \frac{f_K^2}{f_\pi^2} - 3G^\Lambda(\Xi, K) - 3G^\Lambda(N, K) \right] \right. \\ &\quad \left. + D \left[ G^\Sigma(\Xi, K) - G^\Sigma(N, K) + G^\Lambda(\Xi, K) - G^\Lambda(N, K) \right] \right) \quad . \quad (12) \end{aligned}$$

We have retained  $f_K$  and  $f_\pi$ , the difference between which is a higher order effect ( $f_K \sim 1.22f_\pi$ ). However, we know from studies of other observables, such as the octet baryon

<i>Calculation</i>	$\Gamma_{\Lambda\gamma}$ (keV)	$\Gamma_{\Sigma\gamma}$ (keV)	$\kappa_{\Lambda}$	$\kappa_{\Sigma}$	$\mathcal{R}$
Quark Model [3]	143	91	0.25	0.30	-0.15
Rel. Quark Model [9]	118	46	0.22	0.21	-0.20
Bag Model [5]	60	18	0.16	0.13	-0.17
Chiral Bag Model [7]	75	2.4	0.18	0.05	-0.29
Soliton Model (a) [8]	67, 44	29, 13	0.17, 0.14	0.17, 0.11	-0.14, -0.14
Soliton Model (b) [8]	56, 40	29, 17	0.16, 0.13	0.16, 0.13	-0.13, -0.11
$\chi_{PT}$	—	—	$\kappa^{(0)} + 0.078$	$\sqrt{3}\kappa^{(0)} + 0.060 + 0.081i$	-0.075

TABLE I. Radiative decay widths of the  $\Lambda(1405)$  in different approaches. The  $\chi_{PT}$  results are obtained for  $F = 0.4$ ,  $D = 0.6$ ,  $g_{\Lambda^*} = +0.4$  and  $\Lambda_{\chi} = 1$  GeV. We have assumed that the  $\Lambda(1405)$  has vanishing width.

magnetic moments [14], that the additional suppression introduced by the use of  $f_K$  in the kaon loops appears to be present and important. The results of the calculation are shown in Table I. along with the results of other model estimates. We have used the central values for the axial coupling constants ( $F = 0.4$ ,  $D = 0.6$  and  $g_{\Lambda^*} = +0.4$ ) in the estimates from chiral perturbation theory. Using tree-level couplings will increase  $\mathcal{R}$ , as will setting  $f_K = f_{\pi}$ .

Despite the large range of model predictions for the widths and hence the  $\kappa_B$ , (mainly  $\kappa_{\Sigma}$ ) the SU(3) breaking quantity  $\mathcal{R}$  appears to have much less variation. One might hope that this quantity can be estimated reliably enough to make a meaningful comparison with data. We naively estimate that corrections to our results are of order  $\mathcal{O}(m_s)$  if the perturbative expansion is converging. If comparison with data is favorable then the quantity  $\mathcal{R}$  provides a determination of the sign of  $g_{\Lambda^*}$ . We note that the analysis of [10] suggests that  $\mathcal{R} \sim -1.7$ , which is significantly larger than any of the theoretical predictions.

It is also interesting to consider the radiative decay of the  $\Lambda(1405)$  to the decuplet of baryon resonances,  $\Lambda(1405) \rightarrow \Sigma^{*0}\gamma$ . As there is no SU(3) conserving operator (no **1** in  $\mathbf{8} \otimes \mathbf{10}$ ), the amplitude is entirely SU(3) violating with the one-loop graph shown in Fig.2 the formally leading contribution. Writing the amplitude for this process as

$$\mathcal{A} = -\kappa_{10} \frac{e}{M_N} \bar{T}^{\lambda} v^{\alpha} \Lambda^{*} F_{\lambda\alpha} \quad (13)$$

we find the one-loop contribution is  $\kappa_{10} \sim 0.28 - 0.25i$  with the imaginary part arising from the on-shell  $\Sigma\pi$  intermediate state. The fact that the widths of the initial and final state baryons ( $\Gamma_{\Lambda^*} \sim 50$  MeV and  $\Gamma_{\Sigma^*} \sim 37$  MeV) are large compared to their mass difference ( $\sim 20$  MeV) means that we are unable to reliably compute the width and branching fraction for this process. However, to make a very crude estimate of the branching fraction we take the limit of vanishing widths and find a radiative decay width of  $\Gamma(\Sigma^{*0}\gamma) \sim 2.5 \times 10^{-2}$  keV, and branching fraction of  $Br \sim 5 \times 10^{-7}$  (we have used  $E_{\gamma} = 19.9$  MeV). This estimate of the radiative width is substantially smaller than an estimate from the Bag Model [5] of  $\Gamma(\Sigma^{*0}\gamma) \sim 1$  keV. One hopes that these estimates can be investigated experimentally although it is clear that the finite width of each hadron will pose a serious problem.

The question of whether the  $\Lambda(1405)$  is dominantly a  $NK$  bound state or a compact object with a simple interpretation in the quark model is yet to be answered. However, recent work suggests that the  $NK$  bound state picture is closer to the true description [15]. As a small step toward answering this question we have examined the radiative decays of the  $\Lambda(1405)$  in chiral perturbation theory. The  $\Lambda(1405)$  is treated as a fundamental, SU(3) singlet field and incorporated into the chiral lagrangian. This allows computation of corrections to the SU(3) symmetry limit for the radiative decays to the baryon octet. If it is found that our computations are significantly different from what is measured then we will need to examine our assumptions about the  $\Lambda(1405)$ . Firstly, it maybe the case that the  $\Lambda(1405)$  is not entirely an SU(3) singlet and that it may be significantly contaminated with higher dimensional representations, as suggested by the Chiral Bag Model [7]. A not unrelated interpretation of potential deviations is that the  $\Lambda(1405)$  cannot be treated as a point-like field in chiral perturbation theory and does have substantial extent on the scale of the chiral symmetry breaking scale. This leads to a breakdown in perturbative calculability and is consistent with it being an  $NK$  bound state. Therefore, we suggest that measurement of the radiative decays will lead to a better understanding of the structure of the  $\Lambda(1405)$ . It is possible that measurements planned for the near future will be able to determine if the  $\Lambda(1405)$  can be treated as a fundamental, SU(3) singlet field in chiral perturbation theory.

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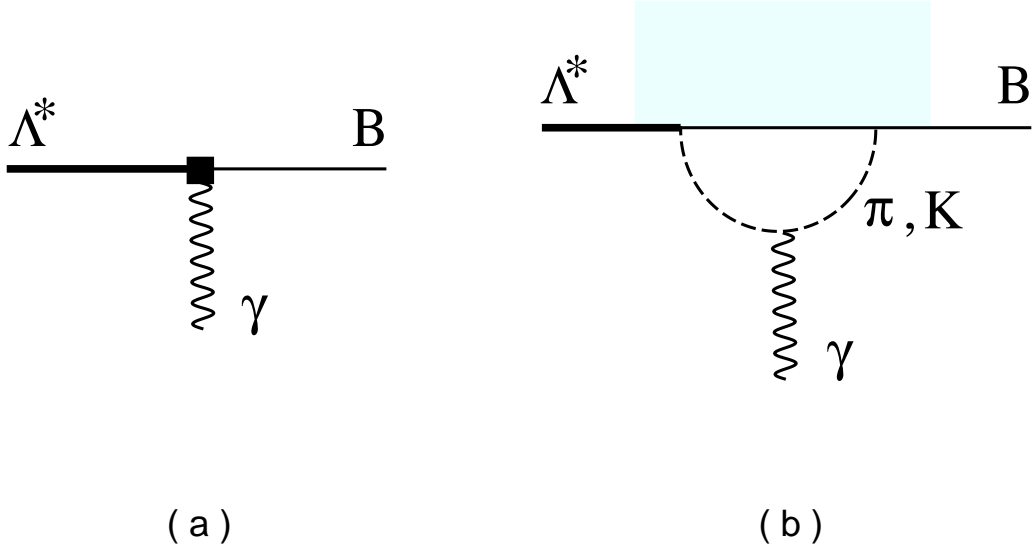


FIG. 1. Tree-level (a) and loop-level (b) contributions to the radiative decay of the  $\Lambda(1405)$  to octet baryons, denoted by  $B$ . The solid square denotes the dim-5 local counterterm. The dashed line denotes a pseudo-Goldstone boson and the thin solid line denotes an octet baryon.

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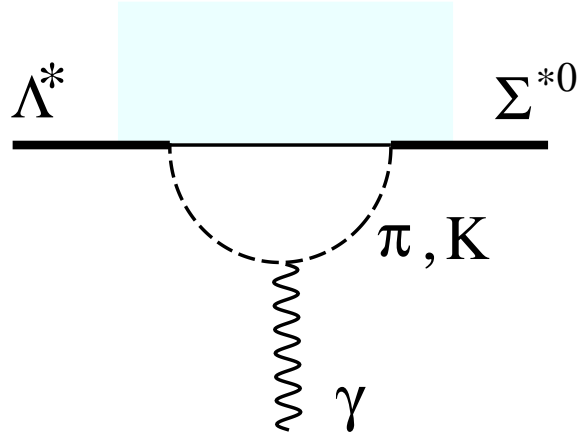


FIG. 2. The loop-level contribution to the radiative decay of the  $\Lambda(1405)$  to the  $\Sigma^{*0}$ . The dashed line denotes a pseudo-Goldstone boson and the thin solid line denotes an octet baryon.